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Growth of a black hole

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Abstract

This paper studies the interpretation of physics near a Schwarzschild black hole. A scenario for creation and growth is proposed that avoids the conundrum of information loss. In this picture the horizon recedes as it is approached and has no physical reality. Radiation is likely to occur, but it cannot be predicted.

Keywords: Black holes; Information loss

1. Introduction

Creation of an eternal black hole – a complete spacetime with the Schwarzschild line element – is a contradiction in terms. We think that it may be possible, nevertheless, to give sense to the notion of "creation of a black hole", more generally "growth of a black hole", and we shall attempt to do so, mainly by introducing an observer, in the established tradition of General Relativity. Here is a rough summary of our proposal.

The sense of "creation" as "becoming in time" is here attributed to an observer who is aware of a limited region of space. Over a period of time he notes that the metric in this region is Minkowski. Then a spherical shower arrives from (spatial) "infinity". In a moment the shower passes the observer, and now he finds himself in a portion of spacetime with a Schwarzschild metric. The further passage of time does nothing to change this metric, except to enlarge the domain on which it applies, unless other showers arrive, each one leading to an increase in the mass parameter of the Schwarzschild metric near our observer.

We consider the idealized case of an isotropic, spherical mass shell of negligible thickness. In contrast to the arrival of a single particle, this event preserves spherical symmetry. The description is also much simpler than the usual treatment of a continuous distribution of in-falling matter. The metric is Schwarzschild (or Minkowski) inside the shell and Schwarzschild outside the shell as well, but with a different mass parameter. Everything is determined by the trajectory, a function f that determines the radius of the shell as a function of time, $r = f(t)$. The Einstein tensor is calculated and we try, without much success, to understand the nature of the matter shell by inspection of this tensor, interpreted as the energy-momentum tensor of the matter distribution.

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In analogy with the famous result on the motion of particles, by Einstein, Infeld and de Hoffman and others, see [6], one might expect that the trajectory is determined by the field equations. The function $1/|\vec{x} - \vec{x}_0(t)|$ associated with a particle is here replaced by the Heavyside function $\theta(r - f(t))$; both lead to the appearance of delta functions in the field equations. We investigate the Bianchi identities, to conclude that no constraints on the trajectory are implied by the field equations. The calculations are not reported, but the conclusions will be supported by intuitive physical arguments.

The conclusion is that the probable trajectory of the incoming spherical shell cannot be predicted from first principles. However, the simple criterion that consists of limiting the velocity of propagation to the velocity of light in the local metric strongly suggests that the infalling matter can never catch up with the horizon of its own making. The fear of Hawking [9] and others, that the matter shell may penetrate its own "shadow", is therefore unfounded.

It is perhaps interesting to try to strengthen the argument by pointing out a simple generalization of the scenario. Imagine a spherical distribution of matter converging towards a point. Instead of trying to solve all the equations, for matter and for the metric, that govern this complicated situation, suppose that matter is concentrated on a large number of concentric spheres. Eventually a continuous distribution of matter may be obtained by passing to a limit. Then between sphere number n and sphere number $n+1$ the metric is sensibly assumed to be of the Schwarzschild type, with a parameter m_n and associated "horizon" $r_n = 2m_n$. Let $R_n(t)$ denote the radial coordinate of the nth sphere at time t. We do not think that it is fruitful to speculate on the equations of motion of each matter sphere, for this would depend on its internal constitution and be subject to wide variation. Grant only that, as long as the density is low, the attraction towards the center, caused by the matter that lies inside, is a dominant factor. Let us admit initial conditions when the density is low and $R_n(t) >> r_n$ and let us suppose that the motion is uniformly inward and radial. It is conceivable that a shell may overtake another shell. It is also possible that shell n may pass through the horizon of shell n'. Remember that the horizon of shell n' is a purely fictitious notion that appears in the analytic continuation of a metric that is physical only in the interval between spheres n' and $n'+1$. It would acquire physical reality only in the event that shell n' caches up with its own horizon, and that we have argued to be very unlikely.

What will happen "eventually"? Well, certainly the most likely possibility is that the inward motion ceases, due to the increasing pressure or by quantum effects. But even if it continues indefinitely there is no difficulty; if the radial velocity does not change sign then it certainly tends to zero, since it takes an infinite time for even a light signal to reach a true horizon.

It might be interesting to extend these considerations to the case of the Kerr metric, for in that case an analysis along the lines of the Membrane Paradigm would give additional insight. We do not know; however, if a probability-conserving quantum mechanics exists in the Kerr metric.

2. The eternal black hole

2.1. Completion of the metric

Throughout this paper the units are fixed so that c (the asymptotic velocity of light), G (the newtonian coupling constant) and $\hbar$ are equal to 1.

Schwarzschild found the first nontrivial solution of Einstein's equations in vacuum [14]. It is static and spherically symmetric and was given by Schwarzschild in the form

$$\mathrm{d}s^2 = (1 - 2m/r)\mathrm{d}t^2 - (1 - 2m/r)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2,$$

with $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2$, for the spacetime

$$-\infty < t < \infty, \quad 2m < r < \infty, \quad \Omega \in S_2.$$

This manifold is not geodesically complete; there are timelike geodesics that reach the horizon at $r = 2m$ at finite proper time though always at infinite values of the coordinate time. The restriction $r > 2m$ was therefore received with scepticism.

Eddington [5], and Finkelstein [7], showed that the singularity at $r = 2m$ could be removed by a change of coordinates. The new time coordinate τ, related to Schwarzschild's coordinates by the formula

$$\tau := t - 2m\ln(r - 2m),$$

leads to the line element

$$\mathrm{d}s_{EF}^2 = (1 - 2m/r)\mathrm{d}\tau^2 + 2r^{-1}\mathrm{d}\tau\,\mathrm{d}r - (1 + 2m/r)\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2.$$

However, this metric fails to be time-symmetric and future directed timelike geodesics run inwards only. (This space really is a black hole!) In fact, the Eddington–Finkelstein coordinates cover only half of a more interesting, time-symmetric spacetime.

To visualize a curved space it is useful to embed it in a flat space of higher dimension. A 2-dimensional manifold can be embedded in 3 dimensions, a 4-dimensional one in 10 dimensions. But the Schwarzschild metric is special, it can be embedded in a 6-dimensional Lorentzian spacetime with signature (5, 1). A partial embedding was found by Kasner [10], the complete embedding by Fronsdal [8]. The complete embedding is given by

$$\begin{aligned}
&\mathrm{d}s^2 = \mathrm{d}Z_1^2 - \mathrm{d}Z_2^2 - \mathrm{d}Z_3^2 - \mathrm{d}Z_4^2 - \mathrm{d}Z_5^2 - \mathrm{d}Z_6^2,\\
&Z_1 = 2\sqrt{1-1/r}\,\sinh\frac{t}{2}, \quad Z_2 = 2\sqrt{1-1/r}\,\cosh\frac{t}{2}, \quad Z_3 = g(r),\\
&Z_4 = r\sin\theta\sin\varphi, \quad Z_5 = r\sin\theta\cos\varphi, \quad Z_6 = r\cos\theta
\end{aligned}$$

where g is the function defined by $(\mathrm{d}g/\mathrm{d}r)^2 = (r^3 + r^2 + r + 1)/r^3$ and the unit of length is $2m$.

The extended spacetime is the surface in 6 dimensions defined parametrically by the equations

$$Z_2^2 - Z_1^2 = 4(1 - 1/r), \quad Z_3 = g(r), \quad Z_4^2 + Z_5^2 + Z_6^2 = r^2. \tag{2.1}$$

It is time symmetric and it is geometrically complete. It can be visualized by fixing the angles so that the metric reduces to $\mathrm{d}s^2 = \mathrm{d}Z_1^2 - \mathrm{d}Z_2^2 - \mathrm{d}Z_3^2$ on the surface

$$Z_2^2 - Z_1^2 = 4(1 - 1/r), \quad Z_3 = g(r).$$

The following illustrations, Figs. 1 and 2, are taken from [8]. Values of r are in units of $2m$.

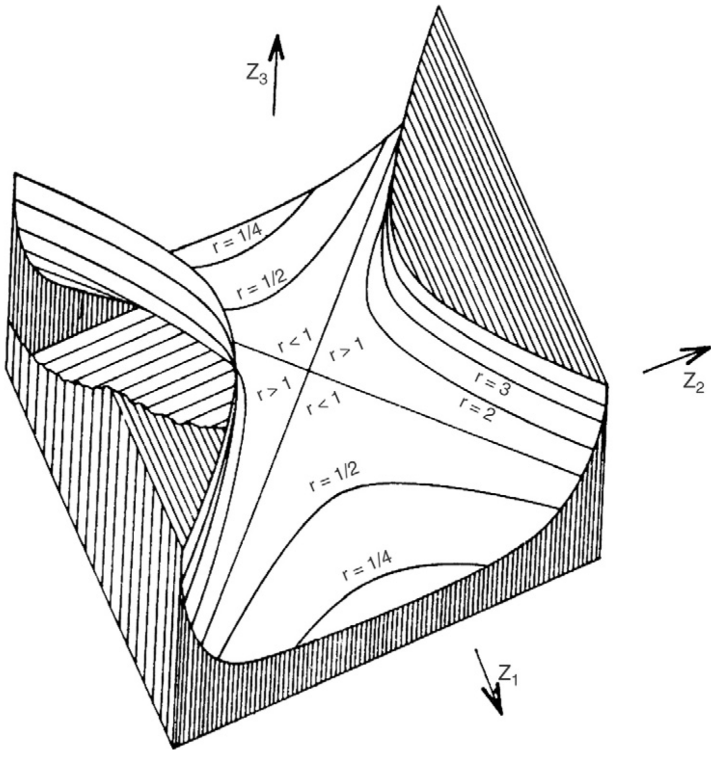

Fig. 1. The surface defined by Eq. (2.1), showing a subspace $\mathrm{d}\phi = \mathrm{d}\theta = 0$ of the completed Schwarzschild manifold as a 2-dimensional surface in a pseudo-Euclidean space.

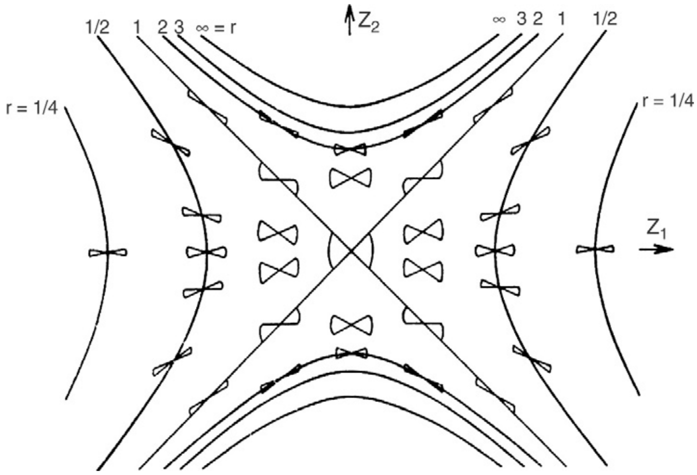

Fig. 2. The projection of the surface in Fig. 1 on the Z_1, Z_2-plane.

The complete manifold contains a pair of time-conjugate Eddington–Finkelstein manifolds. There are 4 distinct regions; two physically equivalent outside regions and two similar inside regions, all of them touching at the intersection of two horizons. See Fig. 2.

Timelike, future directed geodesics originating in the West inside exit into the South outside (=physical spacetime). Some remain there, moving outwards towards infinite distances, others enter the East inside. And some future directed timelike paths come in from infinite distances to either go out again or else plunge into the East inside. (We do not refer to the two insides as "past" and "future", since the entire past and the entire future as we can know them or study them are included in spacetime (the Southern outside)). The appellation "black hole" is no longer apt, for the star is as much white as it is black.

The completion was also found, by other means, by Szekeres [15] and by Kruskal [11].

2.2. *Information and probability in an eternal black hole*

We continue, in this Section 2, to consider the Schwarzschild solution as a given, fixed metric, and the physics of test particles that move in it, without taking any account of the reaction of those particles on the gravitational metric.

A much debated question is the fate of a space traveller who penetrates the horizon, something that seems to be allowed since there are timelike paths that do so in finite proper time.

That is; if we think about an observer in a space ship, for whom the passage of time is measured by the proper time interval $\mathrm{d}s$, then it seems that we must accept that his fate may be to penetrate the horizon and to remain forever inside. The trouble with this idea is that it is not within our rights or duties as scientists to inquire about things that are beyond observation. Imagine a king who sends a more modern Columbus to investigate beyond the horizon. The intrepid traveller dutifully sends back reports of his progress, but these reports arrive back in court with increasing delays. To the king, fixed at home and living in coordinate time, the reports will keep coming,[1] but always more delayed, and always reporting from this side of the horizon. No information from "the beyond" will ever reach the observer that remains outside, the traveller's adventures in the afterlife are beyond human ken and outside the domain with which science is rightly concerned.

Here is a quote from a lecture by Dirac [18] "The mathematicians can go beyond the Schwarzschild radius, and get inside, but I would maintain that this inside region is not physical space, because to send a signal inside and get it out again would take an infinite time, so I feel that the space inside the Schwarzschild radius must belong to a different universe and should not be taken into account in any physical theory". See also Abrams [1] and Antoci and Liebscher [2].

Focusing our attention, as we must, on events taking place outside the horizon, it is perfectly possible to describe such events in terms of the coordinates; more precisely, in terms of r, t and the angles. The completion of the metric

[1] Unless the reports are sent with increasing frequency, traveller time, only a finite number of reports will be received.

serves, not so much to discover the existence of two outsides and two insides, but rather to dramatize the fact that there are two horizons, a future horizon approached by ingoing timelike paths, and a past horizon with outcoming timelike paths. If we limit ourselves to particle-like physical systems, and to their motion along timelike paths, then there is no obstacle to preserving a deterministic physical theory, no loss of information need be feared (or invented). However, to reinforce this conviction we turn to quantum mechanics, for it is a curious fact that quantum mechanics has been subjected to a more extensive discussion of interpretation and information than classical mechanics.

Let us consider the simplest quantum mechanical system on a spacetime with the Schwarzschild metric, keeping an open mind for a moment about the limitation $r > 2m$. The quantum mechanical version of a theory of point particles moving on geodesics is the theory of a scalar field ϕ satisfying the covariant Klein–Gordon equation,

$$\left(\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu + M^2\right)\phi(t,\vec{x}) = 0.$$

With ϕ complex, this comes from the action

$$A = \int \sqrt{-g}(g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi - M^2\phi^*\phi)\mathrm{d}^4x.$$

The probability interpretation of quantum mechanics demands a time independent probability density, and in this case it is given by

$$|\phi|^2 := -\mathrm{i}\int g^{00}(\dot{\phi}^*\phi - \phi^*\dot{\phi})r^2\mathrm{d}r\,\mathrm{d}\Omega.$$

The probability density must be positive, hence $g^{00} = 1 - 2m/r > 0$ and $r > 2m$.

Once more we come to the inescapable conclusion that physics must concern itself exclusively with the outside. There is a great body of work by Chandrasekhar that develops this theory of a scalar field on the outside of the Schwarzschild horizon [4]. It is instructive to recall some of the main features of Chandrasekhar's theory. First of all, he verifies the time independence of the inner product,

$$\partial_t|\phi|^2 = -\mathrm{i}\int \left(\partial_t(g^{00}\dot{\phi}^*)\phi - \phi^*\partial_t(g^{00}\dot{\phi})\right)r^2\mathrm{d}r\,\mathrm{d}\Omega.$$

For a solution of the wave equation this becomes

$$\partial_t|\phi|^2 = -\mathrm{i}\int \left(\phi^*H\phi - (H\phi)^*\phi\right)r^2\mathrm{d}r\,\mathrm{d}\Omega,$$

where H is the "Hamiltonian" operator

$$H = \frac{1}{\sqrt{-g}}\partial_i\sqrt{-g}g^{ij}\partial_j + M^2.$$

Positivity of the probability thus reduces to the condition that this operator must be self adjoint. This fixes the domain of the Hamiltonian, a space of functions that satisfy certain boundary conditions, precisely the condition that $\phi(t,\vec{x})$ vanish on the horizon. The simplest way to demonstrate this is to change variables, replacing the radial coordinate r by the tortoise coordinate of Regge and Wheeler,

$$r^* := r + 2m\ln\left(\frac{r}{2m} - 1\right)$$

This variable runs from $-\infty$ $(r = 2m)$ to $+\infty$ $(r = \infty)$. The inner product takes the familiar form $\int|\psi|^2\mathrm{d}r^*\mathrm{d}\Omega$, and the equation of motion for a stationary state becomes

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}} - V + E^2\right)\psi(r^*,\Omega) = 0$$

with a potential V that falls off as $r^* \to \pm\infty$.

This theory is characterized by a unitary S-matrix and the conservation of probability; there is no loss of information. Incoming waves come from infinity ($r^* \to \infty$) *and* from the horizon ($r^* \to -\infty$), and outgoing waves also approach $r^* = \pm\infty$, as in a one-dimensional scattering problem. This is in full accord with the classical picture.

It needs to be emphasized that this whole discussion, and especially that which follows, is placed in the context of the eternal black hole.

In the ordinary setting of contemporary physics one admits the existence of cosmic rays, of particles and radiation arriving in our midst from distant parts of the universe. It is a useful idealization, in a local context, to consider that cosmic rays originate in the infinite past at infinite distances. There is also the infinite future, populated by the final states of the S-matrix. In a black hole we must admit the existence of an alternative final destiny, the future horizon, the final resting place of particles that do not have enough energy to escape. There is also another possible source of cosmic rays, coming out of the past horizon. We submit that we are no more able to predict this form of radiation than we are able to predict the ordinary cosmic rays.

Although it is probably impossible to predict the arrival of cosmic rays, we can form a scenario that would explain their existence, in the context of a theory of the evolution and even of the origin of the universe. Making any prediction concerning the radiation coming out of the past horizon of a black hole requires a wider context, one that includes the creation of a black hole. This concept of creation of a complete spacetime metric is a contradiction in terms, nevertheless we shall try to make sense of it.

3. Growth of a black hole

3.1. The metric and the Einstein tensor

The proposal advanced here is closely related to the ideas known under the label Membrane Paradigm [16]. However, it will be convenient to postpone comments on this relationship till later.

Let

$$\mathrm{d}s(2m)^2 := (1 - 2m/r)\mathrm{d}t^2 - (1 - 2m/r)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \tag{3.1}$$

the Schwarzschild line element with mass m, on the spacetime

$$-\infty < t < \infty, \quad 2m < r < \infty, \quad \Omega \in S_2.$$

Tolman [17] and others investigated the more general metric defined by

$$\mathrm{d}s_{AB}^2 := \mathrm{e}^{\nu}\mathrm{d}t^2 - \mathrm{e}^{\lambda}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \tag{3.2}$$

where ν and λ are arbitrary functions of t and r, and calculated the associated Einstein tensor $G_\mu^\nu = R_\mu^\nu - \frac{1}{2}R\delta_\mu^\nu$:

$$\begin{aligned}
G_r{}^r &= \mathrm{e}^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2},\\
G_t{}^t &= \mathrm{e}^{-\lambda}\left(\frac{-\lambda'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2},\\
G_\theta{}^\theta &= G_\phi{}^\phi = \mathrm{e}^{-\lambda}\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} + \frac{\nu' - \lambda'}{2r}\right) - \mathrm{e}^{-\nu}\left(\frac{\ddot{\lambda}}{2} + \frac{\dot{\lambda}^2}{4} - \frac{\dot{\lambda}\dot{\nu}}{4}\right),\\
G_t{}^r &= \mathrm{e}^{-\lambda}\frac{\dot{\lambda}}{r}, \quad G_r{}^t = -\mathrm{e}^{-\nu}\frac{\dot{\lambda}}{r}.
\end{aligned} \tag{3.3}$$

By Einstein's field equations this equals -8π times the energy-momentum tensor of all matter present.

The proposed model of mass accretion is based on the following special case of (3.2). Let $0 < r_0 < r_1$ and consider the line element

$$\mathrm{d}s^2|_f = \begin{cases} \mathrm{d}s^2(r_0), & r_0 < r < f(t),\\ \mathrm{d}s^2(r_1), & f(t) < r, \end{cases} \tag{3.4}$$

with $f : \mathbb{R} \to \mathbb{R},\ t \mapsto f(t)$ a smooth function such that

$$\forall t \in \mathbb{R}, \quad r_1 < f(t) < \infty.$$

The idea is to describe a situation in which matter distribution is spherically symmetric and concentrated on a shell, both the inside and the outside regions being empty. This implies that the outside region has a metric that is Schwarzschild, with some mass parameter $2m_1 = r_1$. Recall that the specific form of this line element is derived from Einstein's equation for empty space, which implies that $\nu + \lambda$ is a constant, with the help of a boundary condition that fixes the velocity of light at infinity, which implies that $\nu + \lambda = 0$. On the inside a slight generalization is allowed since the boundary condition does not apply. Till now, we have not found it useful to exploit this possibility and we have thus fixed $\nu + \lambda = 0$. In this case (3.3) reduces to[2]

$$\begin{aligned}
&G_r{}^r = G_t{}^t = (\mathrm{e}^\nu)'/r + \mathrm{e}^\nu/r^2 - 1/r^2,\\
&G_\theta{}^\theta = G_\phi{}^\phi = (\mathrm{e}^\nu)''/2 + (\mathrm{e}^\nu)'/r - \partial_t^2 \mathrm{e}^{-\nu}/2,\\
&G_t{}^r = \partial_t(\mathrm{e}^\nu)/r, \quad G_r{}^t = -\partial_t(\mathrm{e}^{-\nu})/r.
\end{aligned} \tag{3.5}$$

The Einstein tensor vanishes where $r \neq f(t)$, so that matter is present on the moving sphere $r = f(t)$ only. Any and all attributes of this matter that affect the metric are contained in the trajectory function f.

An observer outside the matter sphere experiences a Schwarzschild metric with mass parameter $2m_1 = r_1$, which would imply a horizon at $r = r_1$. But to approach this apparent horizon he must first penetrate the matter sphere at $r = f(t) > r_1$, upon which he finds himself in a region of spacetime endowed with another Schwarzschild metric; the horizon having receded to $r = r_0 < r_1$. In the scenario envisaged here there is no geodesic that connects the outside world to the horizon, a horizon that appears only in the continuation of the outside metric beyond its region of validity. One has a Schwarzschild metric but the horizon is ephemeral and the epithet "Black Hole" is not really appropriate.

The energy-momentum tensor is

$$\begin{aligned}
&(T_t)^t = (T_r)^r = \frac{r_1 - r_0}{8\pi r^2}\delta(r - f(t)),\\
&(T_\theta)^\theta = (T_\phi)^\phi = \frac{r_1 - r_0}{16\pi r}(1 + \dot f^2/A)\delta'(r - f(t)) - \frac{r_1 - r_0}{16\pi A r}\ddot f \delta(r - f(t)),\\
&(T_t)^r = \frac{r_1 - r_0}{8\pi r^2}\dot f \delta(r - f(t)),\\
&(T_r)^t = -\frac{r_1 - r_0}{8\pi A r^2}\dot f \delta(r - f(t)), \quad A := (1 - r_0/r)(1 - r_1/r).
\end{aligned}$$

Notice that the total energy contained in the mass shell is $(r_1 - r_0)/2 = m_1 - m_0$, agreeing with the increase in the Schwarzschild mass parameter.

3.2. The trajectory

We do not possess a convincing theory of a classical matter distribution interacting with the gravitational field. Representing matter by a scalar field leads to the formidable problem of finding interesting solutions of the coupled field equations, though there have been some promising attempts to find exact solutions of field theoretical models that are even more complicated [13]. A point particle may be assumed to move on a geodesic of the field, but we know of no theory of several interacting particles, or of continuous distribution confined to a moving membrane, that satisfies the requirement of invariance under general coordinate transformations. This real difficulty can be avoided by postulating a physically reasonable matter distribution, simple enough that Einstein's field equations can be solved. The first attempt in this direction was that of Oppenheimer and Snyder [12]. Alternatively, but this really amounts to essentially the same thing, one postulates an expression for the metric and studies the implications for the distribution of matter. This can be a very instructive exercise, as shown by the work of Einstein, Infeld and de Hoffman on the motion of several particles [6].

In the context of mass accretion of a black hole it is useful to maintain rotational symmetry. When we postulate a metric that is Schwarzschild everywhere except on a moving sphere we obtain a metric that depends only on the

[2] It is remarkable that the following expressions continue to make sense as distributions although the components of the metric tensor are discontinuous.

trajectory of the matter sphere. The concentration of matter on an infinitely thin sphere is an idealization that is no less physical than the idea of point particles. The trajectory depends on the interaction between the particles and it is not difficult to believe that a smooth trajectory corresponds to physically reasonable interactions.

The simplest example is the static case, $\dot{f} = 0$. The interpretation is especially simple, since the energy-momentum tensor is then diagonal. There is an energy density, radial pressure and transverse pressure. The formulas obtained for the Einstein tensor imply that the pressure (as defined in the manner first suggested by Tolman [17]) is equal to the negative of the energy density. This is unusual, but we are not convinced that it is unphysical, especially since one cannot be sure of what is physical when one is dealing with such a highly idealized situation as is presented by an infinitely thin spherical distribution of matter.

If the internal forces do not balance the gravitational inward pull then the matter sphere will move, inwards or outwards. If the repulsive forces between the particles of the mass sphere are weak or absent we expect a behaviour close to radial geodesic motion. For a test particle with negligible mass in a Schwarzschild metric this would imply, at all times, a relation

$$\epsilon^2 \dot{f}^2 = (1 - 2m/r)^2(\epsilon^2 - 1 + 2m/r),$$

where $\epsilon > 0$ is a measure of the energy. The radial geodesics approach the horizon as t tends to infinity. In our situation a reasonable approximation may be

$$\epsilon^2 \dot{f}^2 = (1 - r_0/r)(1 - r_1/r)\left(\epsilon^2 - 1 + \frac{r_0 + r_1}{2r}\right).$$

This ad hoc formula represents a compromise between what looks like geodesic motion from the inside/from the outside of the matter distribution. One may expect that within a wide family of interparticle dynamics, there will be ingoing trajectories that approach the horizon for large t.

The possibility that the in-falling matter may eventually penetrate the horizon of its own creation seems to us to be extremely unlikely. If we give the mass shell a little thickness we see that a particle on the outside surface is subject to attraction by the mass of the shell, approximately equal to the attraction arising from a star in newtonian gravity. If this attraction would be sustained, then the particle would follow a Schwarzschild geodesic and then it would "never" reach the horizon. In fact, the attraction diminishes as the particle moves through the shell. The particle may be expected, in the absence of interactions with the other particles in the shell, to oscillate within the thickness of the shell, and to fall inwards with a mean velocity that would be less than that of the Schwarzschild null geodesic.

If this argument is accepted then there seems to be no reason to suppose that loss of information is characteristic of physics in the Schwarzschild metric. We have proposed a model of black hole creation and growth in which the horizon is never approached, let alone penetrated.

It is certainly interesting to study a family of reasonable physical models of matter spheres in much greater depth. A statistical approach leads to 2-dimensional thermodynamics, with equations of state and a concept of temperature. In some models the temperature will rise during the inward motion and this would result in radiation. There is already an important literature on this subject [16,13].

The Membrane Paradigm is a very interesting theory of black hole dynamics. The horizon is treated as a physical membrane possessing characteristics similar to a soap bubble with negative surface tension (positive pressure, as required to resist gravitational attraction). In our opinion this is logically possible and not at variance with the correct interpretation of the Schwarzschild solution, for the horizon is the boundary of spacetime and Einstein's equations must be supplemented by boundary conditions. The review by Damour [D] is full of ideas that can be usefully applied within our framework, though it has more to contribute to an understanding of rotating and charged black holes. However, the membrane paradigm is concerned with an already formed black hole; it does not in itself deal with the creation of a black hole, although it stimulates certain ideas about black hole formation [9].

Parikh and Wilczek have proposed [13] to move the membrane out from the horizon, and give it physical reality, and to apply the ideas behind the membrane paradigm to this new situation. Their proposal is quite consistent with our ideas about mass accretion. It may be pointed out that the presence of a membrane effectively hides the region where one expects (when the outside metric is extended too far) to find the horizon. So that Parikh and Wilczek, like ourselves, deal with a Schwarzschild metric but not with a Black Hole.

3.3. Bianchi identities and equations of motion

In the absence of singularities, the (contracted) Bianchi identities,

$$T^{\mu\nu}; \nu = 0$$

are true identities. However, in the presence of a point particle the metric components become singular functions (distributions), such as $1/r$, and meaningless quantities such as $(1/r)\Delta g_{00}$ may appear in the field equations. The discovery that these singularities, dipoles in Einstein's terminology [6], could be eliminated by postulating precise equations of motion for the particles, was a fundamental discovery that set the theory of General Relativity apart from all other field theories. In our context, with step functions $\theta(r - f)$ in the expressions for the metric, one may encounter meaningless quantities like $\theta(r - f)\delta(r - f)$. It occurred to us that the trajectory of the mass sphere may be derived from the requirement that such terms cancel out.

As we pointed out already in a footnote, the calculation of the Einstein tensor does not encounter any untoward singularities. This is not discouraging, for in the case of particles it was not a direct examination of the Einstein tensor that led to equations of motion, but rather the Bianchi identities. Einstein insists on this.

The literature subsequent to the final summary provided by Einstein and Infeld in 1950 [6] is remarkably confused. The Bianchi identities are just that, relations satisfied identically by the Einstein tensor. So if the energy-momentum tensor is identified with the tensor field that satisfies the equation $G^{\mu\nu} = 8\pi T^{\mu\nu}$, then the relation

$$T^{\mu\nu}; \nu = 0$$

is empty. And yet apparently not quite empty! We have attempted to resolve this apparent contradiction as follows. Even if the components of the Einstein tensor are well defined distributions, delta functions in the context of point particles, the same need not be true of the covariant derivative, notably the product of a component of G by a component of the connection, since the latter are not in the Schwarz space of good functions.

This suggests that the elimination of such products should be deemed necessary, and that this is could lead to equations of motion. However, extensive calculations that we abstain from reporting lead to the conclusion that no such thing occurs in our case.

In fact, this result is easily supported by an intuitive argument. The result of Einstein, Infeld and de Hoffman reveals a truth about isolated systems. In order that a point particle deviate from geodesic motion it must have an engine; this implies the emission of some kind of exhaust that contributes to the energy-momentum tensor and violates the initial assumption that this quantity vanish away from the location of the particle. The same conclusion surely applies to a compact and isolated system: If each particle of a compact and isolated system moves on a geodesic then there is no exhaust. However, absence of exhaust does not imply geodesic motion, since the forces may be entirely internal to the system. For example, a membrane may expand or contract due to the existence of surface tension.

Consequently, we can accept the conclusion that no general principle governs the trajectory of the incoming (or outgoing) spherical shower, except for our strong conviction that no reasonable physical assumptions will allow the shower to penetrate the horizon of its own making.

Quantization will alter the picture to some extent. The incoming (or outgoing) massive shell will radiate gravitational waves and these will produce particle pairs. It was suggested that this process can be related to Hawking's proposal [9] concerning radiation from a black hole. But to justify an exclusive preoccupation with this type of radiation requires strong assumptions: (1) that there are no "internal cosmic rays" (see Section 2.2); (2) that one has a complete knowledge of the forces acting within the shell and (3) that the radiation is described by a model quantum field theory. Thus Boulware [3] is able to reproduce Hawking's formula by assuming that there are no internal cosmic rays, no radiation resulting from the interactions between the particles within the shell, and a model of a scalar or spinor field. (This paper [3] considers a thin mass shell but neglects the influence of the mass shell on the metric and this allows us to envisage that the collapsing object – the mass shell – may penetrate the event horizon, which leads to a discussion of "physics" inside the horizon.)

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